**Renumber Test**

Spacetime and Geometry : An Introduction to General Relativity – by Sean M Carroll

By George Keeling, created 24 Feb 2021

Equation numbers (7.1)-(11) have been put at beginning

Equation numbers (1)-(6) have been put at end

Equations (7.1),(8a) do exist

Equation (AB) is referenced three times

Referring to equations (9) and (1) might be risky if they are deleted.

|  |  |
| --- | --- |
|  | (7.1) |

We then found a rank 2 Killing tensor

|  |  |
| --- | --- |
|  | (8a) |

where is the 4-velocity of (all) comoving observers and . We proved it is a Killing tensor in 'Commentary 8.5 Killing Tensor in FLRW spacetime'.

**The Business**

Extending our rule about Killing vectors a bit (Carroll 3.175) we have is conserved along geodesics. is the four-momentum of some (any) particle. Mass is conserved so that means if its four velocity is

|  |  |
| --- | --- |
|  | (9) |

is conserved for the particle along geodesics. Call the conserved quantity and

|  |  |
| --- | --- |
|  | (AB) |

is constant for the particle along geodesics.

**Massive particles (interstellar gas)**

For massive particles and so

|  |  |
| --- | --- |
|  | (11) |

where we introduced where is very like the three-velocity of the particle. Since we are thinking of a particle on a geodesic, it is in free-fall. We also have so (AB) becomes

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |

which is Carroll's 8.101 and shows that particles in free fall "slow down" as the scale factor increases. I suppose the quotes are there because is not quite your usual three velocity which is but the quantity . Also we have not used the true fact that or Carroll's equation 8.100 which was also true. Both seem irrelevant.

As Carroll says free moving particles, such as interstellar gas, slow down. or cool, as the universe expands. Is that surprising?

**Photons**

For photons the line element equation is

|  |  |
| --- | --- |
|  | (3) |
|  | (4) |

where is the four-velocity of the photon. So that's the easy place where Carroll's comes from.

Stick that into (AB) and we get

|  |  |
| --- | --- |
|  | (5) |
|  | (6) |

**Summary**

In section 8.5 we are looking at redshifts and distances. The latter are more complicated than you might think! We start in an FLRW universe with metric